The mean distance from the Earth to the Moon

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The mean value (over time) of the distance between the centres of the Earth and the Moon is 385,000km, not 384,400km as given in many texts. This is based on the Moon's mean equatorial horizontal parallax.

Introduction

What is the mean distance between the Earth and Moon? In this paper, the Earth–Moon distance is defined to be that between the centres of the two bodies; 'mean' herein refers to the mean of this value with respect to time.

Parallax and distance

For an observer on the surface of the Earth, the parallax of the Moon – at a given instant – is the difference between the observed position of the Moon's centre, and the position of this centre for an observer situated at the centre of the Earth. This parallax changes continually by reason of the motion of the observer, due to the rotation of the Earth. It is greatest when the Moon is on the horizon: the 'horizontal parallax'.

Actually, the horizontal parallax is half the angular diameter of the Earth as seen from the centre of the Moon. By reason of the flattening of the Earth's globe, the horizontal parallax of the Moon depends on the geographical latitude. It is largest for a point on the equator: the 'equatorial horizontal parallax'; which is denoted here by EHP.

Because the Earth–Moon distance is continually changing, the Moon's EHP continually changes too. Its value for every day of the year is given in the great astronomical almanacs, such as the *Astronomical Almanac* published in the United States.

From the EHP, the Earth–Moon distance in kilometres can be calculated by means of the following formula:

$$\text{distance} = \frac{6378.14}{\sin(\text{EHP})} \qquad [1]$$

where the numerator is the mean length of the Earth's equatorial radius. What is the mean value of the EHP?

Brown *vs* Chapront

Ernest W. Brown (1866–1938) was an English mathematician and astronomer, who spent the majority of his career working in the United States. The asteroid (1643) Brown is named in his memory.

Brown published in five parts, from 1897–1908, his *Theory of the Motion of the Moon*. This lunar theory has been used, from 1923–1983, for the calculation of the position of the Moon in the *American Ephemeris and Nautical Almanac*, although small corrections were applied in 1954 to produce the *Improved Lunar Ephemeris*.

In the theory of Brown, the Moon's EHP is given by a long series of periodic terms, of which the most important are:

$$\begin{aligned}\text{EHP} = {} & 0.9507245 \\ & + 0.0518128 \cos M' \\ & + 0.0095303 \cos (2D - M') \\ & + 0.0078422 \cos 2D \\ & + 0.0008571 \cos (2D + M') \\ & + \ldots \end{aligned} \qquad [2]$$

The numbers are given in degrees and decimals. M' is the mean anomaly of the Moon – the difference between the ecliptical longitudes of the mean Moon and the perigee – while D is the mean elongation of the Moon; the difference between the longitude of the mean Moon and that of the mean Sun. However this is of no relevance here. What is important is that the angles M' and D increase linearly (uniformly) with time.

Thus, Brown did not provide a formula for calculating the Earth–Moon distance directly. Instead, he gave a formula to obtain the EHP. It is this EHP that is used, for instance, in the calculation of eclipses and occultations. Once the EHP has been calculated for a given instant, the Earth–Moon distance can be found by Eqn [1].

We see that in Eqn [2] the constant term, 0.9507245, is followed by a large number of periodic terms. With respect to time, each of these terms has a mean value equal to zero. Indeed, in the interval 0–360° the mean value of the cosine function is zero. Consequently, the mean value (in time) of the Moon's EHP is equal to the constant term 0.9507245°. From this, the mean lunar parallax Eqn [1] gives a mean distance of 384,399km, which we can round to 384,400km: a value found in most astronomical texts.

In the last years of the 20th century, the French astronomers Michelle Chapront-Touzé and Jean Chapront, of the Bureau des Longitudes, Paris, created a completely new theory for the motion of the Moon: the *Éphémérides Lunaires Parisiennes* (ELP). Here, too, the position of the Moon is calculated by means of a large number of periodic terms. However, contrarily to the theory of Brown, the ELP does not give the parallax of the Moon, but directly gives the Earth–Moon distance in kilometres. The most important terms are:

$$\begin{aligned}\text{distance(km)} = {} & 385{,}000.5584 \\ & - 20{,}905.3550 \cos M' \\ & - 3{,}699.1109 \cos (2D - M') \\ & - 2{,}955.9676 \cos 2D \\ & - 569.9251 \cos 2M' \\ & + 246.1585 \cos (2D - 2M') \\ & + \ldots \end{aligned} \qquad [3]$$

Here, too, the mean value of each periodic term is zero, so consequently the mean value of the Earth–Moon distance is equal to the constant term: 385,000.5584km, which we can round to 385,000km.

This value is 600km larger than the distance deduced from the mean parallax given by [2]. Why this difference, and which one is correct?

Mathematical intermezzo

In my first *Morsels* book, I considered the following mathematical problem.[1] Suppose that the value of a variable quantity A is given by the formula:

$$A = 60 + 2\cos t + 0.3\cos 2t \qquad [4]$$

that is, the constant 60 plus two periodic terms. When t varies from 0° to 360°, the mean value of A is equal to the constant term 60 because, again, the mean value of each of the two other terms is 0. However, let us now suppose that we don't know Eqn [4], and that we have only the following expression for the calculation of the *inverse* of A:

$$\begin{aligned}\frac{1}{A} = {} & 0.01667607303 \\ & - 0.00055325576 \cos t \\ & - 0.00007420329 \cos 2t \\ & + 0.00000261746 \cos 3t \\ & + 0.00000014196 \cos 4t \\ & - 0.00000000885 \cos 5t \\ & - 0.00000000019 \cos 6t \qquad [5]\end{aligned}$$

Then A can be obtained by taking the inverse of the result. By doing the calculation for some value of t, the reader can verify that the formulae [4] and [5] are equivalent. Take, for instance, $t = 33°$; then both formulae should give the same result for A.

Once again, in Eqn [5] the mean value of each periodic term (the terms with cosine) is zero. Consequently, the mean value of $1/A$ is equal to the constant term 0.01667607303, the inverse of which is 1/0.01667607303, or 59.966156, not 60.

The reader might ask why we would use Eqn [5] instead of finding the value of A directly by means of [4]: it is in order to be in the same mathematical situation as is the case for the Moon. Indeed, Brown's theory gives the parallax of the Moon, which is inversely proportional to the distance (this applies only for small parallaxes, but that of the Moon is always a small angle, never larger than 1° 02′). So, with formulae [4] and [5] we have the same problem as for the mean parallax and distance of the Moon.

Solution to the problem

Actually, the solution for the enigma is easy. In our mathematical example, while for a given value of t the inverse of $1/A$ is indeed A, the mean value of $1/A$ is not necessarily equal to one divided by the mean value of A. We have the same situation for the Moon. The mean distance is the constant term in [3], while the mean parallax is the constant term in [2]. Therefore, the mean distance does *not* correspond to the mean parallax. Admittedly, at first sight this seems strange.

The actual reason for the difference between the mean values is that the distance is inversely proportional to the parallax. Here we suppose that the parallax is a small angle, say, smaller than 1° or 2°. The smaller the parallax, the larger is the distance. But for a given variation of the parallax, its effect on the distance is greater for a large distance than in the case of a smaller one.

Let us illustrate this by means of a fictive example. Let D be the distance between two points, given by $D = 210/P$, where P is the parallax (the units used here are irrelevant). Suppose that during half of the time P is equal to five, and that during the other half of the time it is equal to seven (of course, this is not realistic because a parallax cannot suddenly jump from five to seven, but here it is just a mathematical example). The mean value of P is six, and therefore we might think that the mean distance is equal to 210/6 = 35.

However, for $P = 5$ we have $D = 210/5 = 42$. For $P = 7$, the value is $D = 210/7 = 30$. Consequently, the mean distance is (42+30)/2 = 36, not 35. Therefore, the mean distance (over time) is larger than the value we would deduce from the mean value of P.

Conclusions

The mean Earth–Moon distance (the average over time) is 385,000km, not the 384,400km that we read in most publications. The latter value is what we would deduce (incorrectly, as we have seen) from the Moon's mean parallax.

It happens – quite accidentally(?) – that 384,400km (give-or-take one or two kilometres) is also the length of the semi-major axis of the lunar orbit. However, this is *not* equal to the mean distance, as discussed below. Murphy (2013) states it clearly:[2] 'while the semi-major axis of the lunar orbit is 384,402km, the time-averaged distance between Earth and Moon centres is 385,000.6km.'

In this regard, it might be of interest to note that, for an elliptic orbit of a planet, the semi-major axis is often called the 'mean distance' to the Sun. We even read this in some official publications,[3] but this practice is just one of convenience; actually, the true mean is the average over time.

This is readily apparent from an extreme case: the periodic comet 1P/Halley. The semi-major axis a of its orbit is approximately equal to 18 astronomical units (AU). However, the comet remains near its far aphelion for a very long time, at about 35AU from the Sun, where it moves slowly. The comet remains close to the Sun for only a short time near the perihelion of its orbit; because there its speed is very large. So, it is quite evident that for this comet the mean distance to the Sun (the average *over time*) is much larger than $a = 18$AU.

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